A Hybrid Pricing Expert System Based on Fuzzy Reasoning

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Abstract: *Regardless of the increasingly more sophisticated and precise mathematical models formulated in the field of economics, a persistent gap remains between the predictions and the economic reality. The uncertainty and complexity of the involved phenomena fails to be encapsulated into "precise" models. With the most important part of the related information stemming from human experts, it is more than reasonable to employ models that are capable of hosting and reasoning with such knowledge. It is argued that the fuzzy logic and fuzzy reasoning framework is perfectly suited for such an approach since it provides the required expressiveness for the knowledge representation and the respective inference mechanisms to efficiently deal with the pricing problem. This argument is discussed and verified with the use of the FUZZYPRICE system that is designed and implemented to assist pricing decision-making. The results obtained are then compared to the well known PRICE-STRAT system to demonstrate the effectiveness of the current approach.*

Keywords: Pricing decision-making, price-wars, expert systems, fuzzy logic, fuzzy reasoning, car pricing.

1. Introduction: The Pricing Problem

Pricing has always been one of the primary issues a business has to take care of, considering the fact that setting the appropriate price has an immediate effect upon the company's revenues and profitability.

1.1. Substantial Factors

For pricing decision-making numerous parameters should be taken into consideration. First, the company has to decide which are the *strategic aims* of its pricing moves. These can be survival (under intense competition or overcapacity), maximization of current profits, maximization of current revenue, maximization of sales growth, maximization of market skimming (i.e. lowering prices periodically in order to capture more than one market segments from the less price-sensitive to the ones who consider low prices a primary issue), and product-quality leadership.

It is more than often that a company is forced to sell its products at prices which lead to profit minimization or even short-term loss. Intense competition often forces a company to follow this survival-oriented decision-making. In an effort to gain market share, competitors lower their prices. If this situation is sustained for a prolonged time period, even the most conservative firms comply with the prevailing strategy. Overcapacity is another reason for a firm to sell at cost, since an overgrown stock may lead to liquidity problems.

- *Profit maximization* (long-run or short-run) is, usually, a primary concern of business managers. Pricing products towards this direction involves complicated decisions, since one should consider the price-demand schedule, competition, and exact evaluation of costs. Especially, current profit maximization may lead to wrong marketing decisions, if the financial status of the company is neglected, or if state laws are not considered thoroughly.

  When current revenue maximization is the issue, it is absolutely necessary that the demand schedule is evaluated. Very often, managers consider this strategy as a way to reach long-run profit-maximization. On the other hand, maximizing sales means lowering prices substantially in order to gain a greater market segment, and creating the “environment” for a potential profit-maximization. Sales maximization can be accomplished if the market is price-sensitive.

- *Quality* is an other key factor, since companies determine different policies combining different degrees of quality with different price levels in order to accomplish their strategic aims. Through high quality, a product can evolve into a market-leader and force the competition to follow the company's pricing policy. Generally, achieving the best price-quality combination is a matter of critical importance and increased complexity.

- Marketing managers always try to determine the *market-demand schedule*. Obviously, the higher a price is set the lower the sales volumes are, but the exact shape of this curve and the applicability of this law to various products are rarely easy to determine. Estimating demand schedules and the price elasticity of demand involves answering these questions in an accurate way. Once price-demand relation is figured out pricing becomes a much easier task to perform. However, it is impossible to find an exact and accurate solution to this problem. Usually, managers rely on their experience and instincts, aided by market research analysis, historical data analysis and experiment analysis methods. One can always be certain of what happened in the past and of the specific ways public reacted to the specific events (price changes, political-economical, and other), but one can never be certain about the future.

- What companies can always be accurate on is estimating the *cost structure* of a product. This is usually the first action performed in order to select the selling price. Company costs set the floor in pricing decisions. There are two types of costs: variable costs and fixed costs affecting the final price policy in different ways. Marketing managers also take into consideration cost behavior at different levels of production per period and cost behavior as a mathematical function of accumulated production.

- The last, but in no way least, of the factors significantly affecting pricing decisions, is *competition*. Considering the basic competitors' prices and quality standards, a company is able to determine selling prices in order to follow competition or to overcome it when that is possible. Demand and cost structures set the higher and the lower prices, but competition orients the exact pricing decisions. Competition's importance depends on the type of the market. Pricing can evolve into a most powerful weapon in fully antagonistic markets, whereas in oligopolistic ones, it might be of minor importance. More than often the case is not that clear. More than one basic competitors are involved in the "pricing game", and the company is forced to alter the selling prices so that greater market segment can be achieved, or profit can be maximized, or the "image of the product" can be sustained. This situation describes a "price war".

1.2. Fundamental Pricing Methods

From the above parameters some are more influential than others, depending on the specific requirements of each pricing case.

A company may choose a cost-oriented policy, which is the oldest and easiest to apply, leading to the so called *mark-up pricing*. A premium to the product's cost is estimated and the price is set. This method is risky to the extent that it neglects the other influential parameters (demand, competition, etc.) Furthermore, companies often overprice new products in order to cover their costs as quickly as possible and lose in this way market segments of vital importance. Great advantages of this method are simplicity to apply and justice served to the customers.

A slight variation to this technique, called *target-return pricing*, involves accumulating the desired preset profit little by little. According to this method the initial investment is covered in a certain and pre-calculated pattern, following a plan for a period of time. The method is usually implemented by Public Benefit Corporations, because in these cases demand can be forecasted.

Perceived-value pricing sets the selling price according to the buyers' view of the product and *going-rate pricing* is a strictly competitor oriented pricing method ("follow the leader").

The above methods are only generalized policies. In the final decision, parameters as geography, special company policy, psychology, product-mix, discounts and rebates, and others are of major importance.

1.3. "Price Wars"

Initiating a price change is always a risky decision, since competitor and buyer reactions are rarely the most expected ones. Studying the patterns of these reactions and carefully pre-calculating them is a primal issue of modern pricing techniques. The enterprise must also anticipate the probable reactions of suppliers, middlemen, and government.

The firm facing a price change initiated by a competitor must try to understand the competitor's intent and the likely duration of the change. If swiftness of reaction is desirable the firm should pre-plan its reactions to different possible price actions by competitors. An engagement into a “price war” is a constant situation, especially in short-term periods, forcing a company to fight its competitors solely by pricing means.

The competitors are most likely to react in markets where the number of competitors is limited, the product is homogenous and the clients are well informed. In order to win a “price war” a company must be well aware of the opponents’ advantages and disadvantages concerning the financial status, the current sales, the production capability and the brand loyalty by the customers. Furthermore the competitors’ goals are of vital importance in order to guess the future price-quality moves. The time element is critical in price changes as well as the possible impact. Finally, decisions during a price war vary according to the position in the market the company holds. For example, market leaders may follow an aggressive pricing policy when “attacked” by smaller brands, or may chose to keep their image if the potential danger is limited.

1.4. Assisting Tools for Pricing

Several tools such as Decision Support Systems, Expert Systems and Models have been designed and implemented to assist pricing decision-making during price-wars. Their use became gradually obligatory from optional because of the complexity of the matter and the limited capabilities of unaided human intelligence in dealing with the vast amount of involved parameters, data, and knowledge. Some of these systems, either for general pricing or applied to specific products, are listed below:

1. PRIDEM (modeling competitive pricing and market share) [1]
2. Non-parametric approach to pricing and hedging derivative securities (based on learning networks) [2]
3. PROMOD III, ELFIN (based on production cost models) [3]
4. Pricing initial public offerings (based on neural network models) [4]
5. Services pricing expert systems (based on differential premiums and cost premiums) [5]
6. Pricing automobile insurance under a cross-classification of risks [6]
7. Pricing strategy based on Bayesian decision theory [7]
8. PEMS (Pricing for profits) [8]
9. Loan pricing (a microcomputer model) [9]
10. PORTFOL, DATAIN [10]
11. Transportation Backhaul pricing (based on neural networks) [11]
12. Pricing of oil and gas [12]

PRICE-STRAT [14] appears to be one of the most promising models. It is a hybrid expert system combining both historical data and human experience, that is able to derive the most suitable portfolio pricing solution for various industries, as for automobile and gasoline, or for banking products and airline tickets. The focus of the system is the best reaction to every competitor's move, even during price wars.

The system's methodology involves the following steps. First, a managers' committee is gathered and questioned in order to establish the strategic goals and constraints of the company. Then, PRICE-STRAT generates a series of “what-if” scenarios concerning the possible sales variations in different prices of the company's and the competitors' products. Each manager fills out every single one of them and they are then fed into the system. Thus, the knowledge base is created. Historical data and Market Research data are added and the system's base is ready for questioning. An Adaptive-Predictive model is then implemented. It derives the most suitable for the case price solution among the various solutions proposed through the knowledge base questioning. Variable and fixed costs, price and volume constraints are all taken into consideration.

2. Fuzzy Logic: An Overview

2.1. Motivation for Fuzzy Logic in Economics

Regardless of the increasingly more sophisticated and precise mathematical models formulated in the field of economics during the last century, a persistent gap remains between the predictions and the economic reality. Several reasons could be identified for this [15], the most important of which are discussed below.

While the present mathematical models are based on “classical” mathematics, the human reasoning and decision making, which play a dominant role in economic phenomena, is strongly affected by the uncertainty embedded in natural language which fails to be encapsulated into such “precise” models.

The continuously increasing complexity of economic phenomena calls for continuously more complex mathematical models. When this complexity becomes unmanageable *simplifications* are in order. But by simplifying, one actually restricts the scope of the model under construction which implies an attempt to *precisely solve an approximation of the original problem*. Practical applications have proven that in certain cases, the performance of such approaches may be significantly lower than that of an approach which *approximately solves the original problem* like the one undertaken by fuzzy logic.

For most problems, two sources of information are present, namely *numerical information* in the form of measurements of important model variables and *qualitative information* coming from the human experts in the form of commonsense linguistic propositions. While conventional methods can only make use of the first kind of information, fuzzy logic provides the means to efficiently host both under a single uniform system shell. Here systematic knowledge acquisition techniques under uncertainty must be applied [16].

In the following a short overview of fuzzy concepts will be provided.

2.2. Fuzzy Sets and Fuzzy Operations

A central notion in mathematics is the *set*. The definition of a *crisp set A*, on a universe of discourse *X*, is based on the premise that an element of *X* either fully belongs to *A* or not. *Fuzzy sets* emerge by allowing partial membership of an element to a set [17].

Given two fuzzy sets, *A* and *B*, of the universe of discourse *X* having membership functions $\mu_A(x)$ and $\mu_B(x)$ respectively, the following operations may be defined:

Complement $\overline{A}$: $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$

Union $A \cup B$: $\mu_{A \cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$

Intersection $A \cap B$: $\mu_{A \cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$

2.3. Linguistic Variables and Hedges

Intuitively, if a variable can take words in natural language as its values, it is said to be a *linguistic variable*. These words are actually labels of fuzzy sets.

Linguistic hedges are special linguistic modifiers. For example, for a fuzzy set F (with membership function $\mu_F(x)$) that captures the idea of "young", some intuitive linguistic hedges are defined as:

very(F): $\mu_{very(F)}(x) = (\mu_F(x))^2$

more-or-less(F): $\mu_{morl(F)}(x) = \sqrt{\mu_F(x)}$

2.4. Fuzzy Rules

The general form of a (MISO Mamdani-type) fuzzy rule with n inputs, x_i : i=1...n, and a single output, y, has the following form [18]:

IF x_1 IS F_1 AND ... AND x_n IS F_n THEN y IS G

where F_i and G are fuzzy sets, while x_i and y are linguistic variables. Any multi-input multi-output (MIMO) rule can be decomposed into a set of such MISO rules which are general enough to express many forms of linguistic knowledge (incomplete antecedent part, OR rules, statements, UNLESS rules, crisp rules, etc.).

An alternative form of fuzzy rules was suggested by Sugeno [19]. A typical *zero-order Sugeno fuzzy rule* has the form:

IF x_1 IS F_1 AND ... AND x_n IS F_n THEN $y = p$

where p is a crisp constant. Although Sugeno-type rules share many common characteristics with Mamdani-type rules a major difference is that the descendent of the latter are crisp. Similarly, a more useful class of Sugeno rules is defined, the so-called *first-order Sugeno fuzzy rules* which have the form:

IF x_1 IS F_1 AND ... AND x_n IS F_n THEN $y = p_1x_1+...+p_nx_n + p_0$

where p_k , k = 0...n are constants. Higher order Sugeno type fuzzy rules are also possible but introduce significant complexity.

Sugeno type rules are more compact and computationally more efficient than Mamdani type rules and naturally lend themselves to adaptive techniques [20]. On the other hand, Mamdani type rules are more intuitive and better suited for human input.

2.5. Assigning Membership Values

A key issue that emerges from these forms of fuzzy rules is the way the membership functions of the antecedents of the rules (the fuzzy sets F_i), the descendent parts of the rules (the fuzzy set G for Mamdani, and the constants p_k for Sugeno type rules), as well as the rules themselves are determined.

The literature is rich with references on the topic of assigning appropriate values to these 'model parameters'. The various methods may be divided into two categories, namely (i) methods that rely on human experts, and (ii) training algorithms based on exemplar data. These methods usually address both the way the rules and the rules' fuzzy sets and parameters are obtained. A draft and by no means complete list of the most important ones follows:

- *Intuition,* which relies on the innate intelligence of human domain experts as well as to commonsense understanding of the problem at hand.
- *Rank ordering,* which assigns membership function values by assessing preferences by a single individual, a committee, a poll, or other opinion methods [15].
- *Neural networks*, have been proposed to determine membership functions based on sample data and clustering techniques [25].
- *Genetic algorithms,* may also be used as long as a fitness function is formulated to evaluate alternative membership functions [26].
- *Adaptive fuzzy system techniques* [21], as backpropagation, orthogonal least squares, nearest neighborhood clustering, table lookup schemes, etc.

It is important to note that assigning membership values to the fuzzy sets involved in fuzzy rules is a twofold problem consisting on both determining their functional forms (triangular, trapezoidal, Gaussian, etc.) as well as their parameter values (e.g. the center value and the spread for a Gaussian membership function).

3. Fuzzy Expert Systems and Fuzzy Reasoning

An expert system may be roughly described as a computer-based system enhanced with a knowledge based component in such a way that it may yield an 'intelligent' advice or decision. This is achieved with the aid of a rule-based inference scheme. Expert systems have been useful in a variety of real-life applications from engineering, to medicine, to space and management [22, 23]. Fuzzy expert systems may be considered as an extension of the classical expert systems in that they may represent and deal with uncertainty using fuzzy rules and fuzzy information processing. The architecture of a fuzzy expert system (FES) is shown in Figure 1.

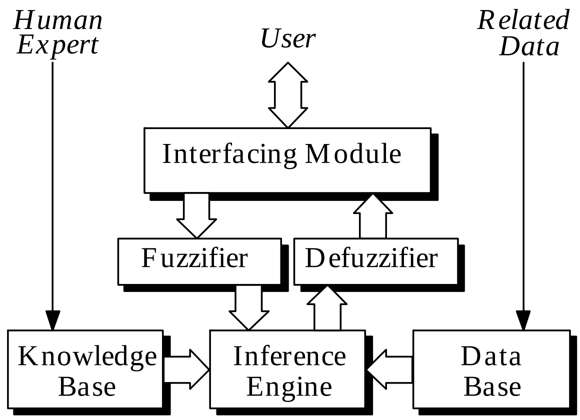

Figure 1: *Architecture of a fuzzy expert system*

The architectural components of the system are:

- The *knowledge base*, containing fuzzy production causal rules of the form "IF *A* THEN *B*", *A* and *B* being fuzzy sets defined on the input and output domains respectively, as well as the definitions (membership functions) of the possible values of *A* and *B*.
- The *database*, used to store parameters of the problem or other related facts.
- The *inference engine*, responsible for evaluating the fuzzy rules of the knowledge base (typically using generalized modus ponens or generalized modus tollens).
- The *interfacing module*, responsible for obtaining inputs from and providing outputs to the user.
- The *fuzzification* and *defuzzification modules* (fuzzifier and defuzzifier), which convert a crisp input to a fuzzy set and a fuzzy set to a crisp output respectively.

Often, a *metaknowledge base module* is used to represent knowledge relative to the rules of the knowledge base (e.g. supervisory knowledge, rule modification knowledge, etc), and an *explanatory interface module* to supply the user with details on how the expert system reached a certain answer.

Fuzzy reasoning using this fuzzy expert system consists of the following steps:

- Fuzzify the crisp input obtained by the user, say *a'*, to produce a fuzzy set *A'* using one of the available fuzzification methods (singleton, Gaussian, etc) [24].
- Evaluate all rules of the knowledge base using fuzzy implication (e.g. generalized modus ponens rule, Zadeh's min-max rule, etc). This is accomplished by calculating the consistency of *A'* with the *i*-th rule's antecedent A_i. Based on this and the descendent of the rule, say B_i, calculate the respective output of the *i*-th rule, say B_i'.
- Aggregate the outputs of all rules to form the overall output, say *B'*.
- Defuzzify the fuzzy set *B'* to produce a crisp output, say *b'*, to be presented to the user. Any of the proposed defuzzification methods can be used depending on the problem at hand (maximum, center average, modified center average, etc.)

When the fuzzy knowledge base consists of Sugeno-type rules the only difference in the above procedure appears in the aggregation and defuzzification steps which are replaced by a simple weighted averaging of the crisp outputs of all rules. This fact greatly simplifies the required computations.

A thorough survey on fuzzy expert systems can be found in [16].

4. Learning and Adaptation in Fuzzy Systems

For a system, learning roughly means adjusting its free parameters so as to minimize an error criterion or cost function [24].

While *learning* usually refers to the process of adjusting parameters *off-line* and prior to setting the system in operation, *adaptation* refers to their *on-line* tuning so that the system tracks a temporally changing target behavior. However, this distinction is quite 'soft' since the same algorithms and techniques are often used for both procedures.

Learning may take place to systems based on either Mamdani or Sugeno type rules. For systems based on Mamdani type rules the adjustable system parameters are the parameters of the fuzzy sets of the antecedent and descendent parts. For systems based on Sugeno type rules, the adjustable system parameters are the parameters of the fuzzy sets of the antecedent and the constants of the descendent part of the rule.

Various algorithms are available in the literature which are actually based on system identification, optimization, optimum filter design, etc. Since the minimization of a cost function is the heart of such algorithms, neural techniques prove to be particularly useful [24]. In the simplest case, the well known *backpropagation algorithm* may be naturally embedded into a fuzzy system providing a straightforward, easy to implement method for adjusting the parameters but at the cost of slow convergence and the possibility of getting trapped at local minima and sub-optimum solutions. Other methods that lend themselves to parameter adjusting include the least mean square algorithm, the Newton-Gauss method, Levenberg-Marquardt algorithms and others.

In both Mamdani and Sugeno system types various methods have also been proposed (e.g. genetic algorithms) to on-line delete 'inadequate' rules and replace them with automatically formulated new rules whose performance is to be evaluated during the operation of the system.

In conclusion, fuzzy systems are mainly based on linguistic knowledge in the form of fuzzy rules, while adaptation techniques make use of numerical information obtained by measurements or observation. Thus, adaptive fuzzy systems are capable of hosting both types of information (as opposed to most other engineering systems) and prove to be both efficient as well as easier to design and bootstrap.

5. The FuzzyPrice System

The proposed system, called FuzzyPrice, makes use of fuzzy logic and fuzzy reasoning techniques to calculate appropriate combinations of the portfolio prices of a company's products, which are expected to best serve the strategic aims of the company while respecting its strategic constraints. The inference is based on the price changes of the competitors in the presence of price wars. A point to be stressed is that the system's applicability is by its nature restricted to short or midterm and not to long term pricing.

5.1. What-If Scenarios

One of the key characteristics of FUZZYPRICE is that its operation is based on *fuzzy 'what-if' scenarios*. These scenarios are the heart of the inference procedure of FUZZYPRICE and are used to relate perceived changes in the competitor prices to changes in the sales of the respective products. A set of such scenarios may, in effect, be used as a form of a 'market predictor'. The similarity of the structure of a what-if scenario and a fuzzy IF-THEN rule is noticeable which is the reason why the fuzzy what-if scenarios where implemented via fuzzy rules in FUZZYPRICE.

For example, consider the case of a company selling a product in market with two competitors. A price war could emerge when the other two competitors alter the value of their products. A fuzzy what-if scenario for that case, which at the same time is also a fuzzy rule, may have the form:

IF *dPrice1* IS *PNegativeHigh*
AND *dPrice2* IS *PNegativeLow*
AND *dOurPrice* IS *PZero*
THEN *dOurSales* IS *SNegativeMedium*

where *dPrice1* and *dPrice2* are fuzzy linguistic variables capturing the changes of the prices of the product of the first and the second competitor respectively, *dOurPrice* and *dOurSales* are fuzzy linguistic variables capturing the change of the price and the change of the sales of our product, and *PNegativeHigh*, *PNegativeLow*, *PZero*, and *SNegativeMedium* are fuzzy sets capturing a high decrease in the price, a low decrease in the price, almost no change in the price and a high decrease in the sales respectively.

It is clear that such what-if scenarios are both very expressive and easy to formulate by human experts. The question on how to define appropriate fuzzy sets for the antecedent and the descendent of the rule was treated on a previous section and mostly relies on the intuition of the company's managers.

Several scenarios of this form may be inserted and stored into the system. In the case of an actual price war fuzzy inference techniques provide a straightforward way to fire the relevant scenarios in parallel and merge their output to produce a valid estimate of the change of sales of the company's product under the specific circumstances at hand. This way an efficient interpolation takes place for the case of inputs that are not explicitly present in the what-if scenarios.

5.2. Intra-Portfolio Side-Effects

It is clear that for every product of the portfolio, a different set of what-if scenarios, i.e. a different fuzzy system need to be obtained to capture the special characteristics of the market group the respective product addresses. An important feature that fails to be incorporated in this scheme of distinct fuzzy systems is the '*intra-portfolio side-effects*' of changing a product's price, i.e. the effects of changing the price of one portfolio product to the sales of another portfolio product of the company of interest. For example, when lowering the price of a 1400cc car and increasing the price of a 1000cc automotive could, under certain conditions, lead a market group to shift from the 1000cc to the 1400cc since this may be perceived to have the best money to value ratio.

So, it is clear that a precaution need to be taken by the system so that these intra-portfolio ‘side-effects’ are taken into account. Again, a reasonable candidate for such a module is a fuzzy system with appropriate fuzzy rules to predict the effects of changing a product price to the sales of the *other* portfolio products. Although arbitrary, this approach is both intuitively appropriate and in accordance to the overall fuzzy framework of FuzzyPrice. In this module too, the fuzzy rules may also be viewed as what-if scenarios.

5.3. Embedding Strategic Aims and Restrictions

The ‘best price’ for a company’s portfolio depends on various factors, the most important being the company’s strategic aims at the time of the decision and its strategic constraints. The *strategic aims* may be different at different times. For example it may be survival under intense competition, it may be current profit maximization, or main concern may be to increase the sales and/or the market share. Moreover, various strategic constraints need to be satisfied which may be imposed on:

- *the minimum and maximum amount of sales*, depending, for example, on the minimum market share desired and the production ability of the company respectively,
- *the minimum and maximum product prices and their difference to the competitors’ prices*, depending on the product market image desired for each portfolio product, the production cost, the minimum profit, etc.,
- *the minimum and maximum differences among the prices of the products of the portfolio*, depending, for example, on the current market segmentation.

5.4. Market Research and Historical Data

Although one of the most important sources of information in managerial decision making is the human expertise coming from the company’s managers in the form of qualitative knowledge, other sources of information may also provide valuable data. Such sources are:

- *Market research data*, which provide a picture of the market preferences, desires, needs, etc. The main disadvantages of this kind of information is its short time value which quickly degrades, and its relatively low accuracy that strongly depends on the methods used to obtain them.
- *Historical data*, which provide a clear picture of past causal relations of company price moves and respective market responses. This kind of information also suffers from degradation of quality with time.

The main characteristic of these two sources of information is that they usually provide data of numerical nature. Such data are extremely important since they are facts and not prediction and because they can be immediately used by an adaptation algorithm to refine the performance of the system by adjusting the parameters of our model, i.e. the parameters of the fuzzy knowledge bases as described in a previous section.

But this is not the only update that the knowledge bases need in order to remain consistent with the real market that they try to predict. A routine procedure that needs to be performed to keep the knowledge bases up to date is the partial editing of its rules by human expert in order to make changes of points of view of one or more managers, introduce new ideas and trends that are observed in the market, eliminate rules that become obsolete, etc. Much of the system's performance depends on such procedures.

5.5. Sub-Optimal Solutions

In pricing as well as in most complex decision making processes the role of human decision makers is and will remain essential. The final decision among several alternatives is still selected by managers and directors. The role of a decision support system is to exhaustively search numerous alternatives, end up with just a few that are noteworthy, and present them to the user. This feature is respected in FUZZYPRICE where a number of alternative 'sub-optimal' solutions are also obtained, leaving the final choice to human decision makers.

5.6. The Overall System Architecture

Figure 2 illustrates the architecture of the FUZZYPRICE system. The system consists of a number of fuzzy expert systems working in combination with simple computational blocks. For each of the products of the company at hand, the price moves of all the respective competitors and of the company itself are input to the system. Assuming *n* products in the portfolio, a layer of *n* independent fuzzy expert systems, undertake the role of 'predicting' the effect of these moves. Each of the fuzzy expert systems contains specialized rules to capture the special market characteristics of the respective products and to decide (on the basis of the what-if scenarios that have been built in) the effect of these moves upon the sales of the specific product.

For a specific move of the competitors, a set of possible reactions is automatically produced and fed to the fuzzy expert systems layer. All the respective outputs are stored to temporary tables and are subject to the price and sales restrictions imposed by the company's strategic aims. This way some of the solutions need to be rejected since they might lead an increase to the sales that does not conform with the production capabilities of the company, or they may require the price to get off the desired interval, etc.

Up to this point, each product is examined independently from the others. In order to compensate for the intra-portfolio effects, all these independent scenarios need to be merged into complete overall scenarios. The last fuzzy expert system perform the required compensation. From the final table of all the possible moves and respective revenues, it is straightforward to select the ones that closer serve the strategic aims of the company, e.g. the ones that maximize the profit, the sales, etc.

A small set of the 'best' solutions can be presented to the human manager who has to take the final choice for the actual move of the company.

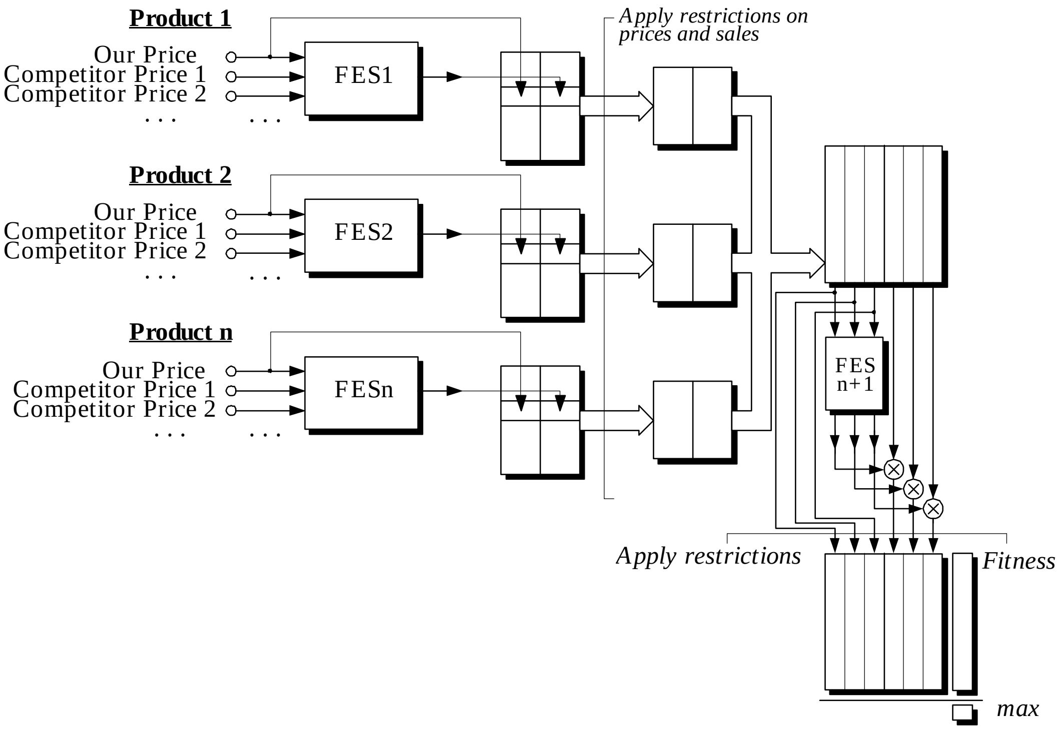

Figure 2. *The overall system architecture*

6. Application Example: Simulation Results

6.1. The Application Problem

To evaluate the performance of the FUZZYPRICE system, the problem of pricing in automotive industries is addressed. The objective is to set the price for three car models of a company so as to maximize the total expected short term profits. Different competitors are present in each car model. The system inputs are the changes of the prices of the competitors products for each of their respective models. The desired system output is a set of values for the three car models that will, hopefully, maximize our profits.

The specifications of this problem were chosen to be as close as possible to a similar problem treated by Singh [14] so that useful comparisons can be made between FUZZYPRICE and PRICESTRAT which is a system tested under real conditions and proven to be quite successful.

The three car models to be investigated were chosen to be 8v, 16v, and Turbo 4×4 while, for simplicity, the number of competitors for each model were chosen to be 2, 3, and 2 respectively and given in Table 1.

Table 1: *Competitor products for each category*

8v	16v	4×4
Mazda MX-3 1.8	Toyota Celica 2.0 GT	Toyota Celica GT 4WD
Toyota MR2 2.0	Toyota MR2 GT	Audi Quattro Coupe 2.8
	Nissan 200SX	

Additional tables containing more information on these car models as well as information on the models of the company of interest, can be found in the paper by Singh [14]. The strategic constraints are also present there and concern the higher and lower limits for the prices and sales of our company as well as the differences among our model prices with the respective competitor prices and with themselves.

It should be noted that no assumptions are made concerning the sales of the competitors products.

6.2. System Structure Details

Three fuzzy expert systems where designed, one for each car model, having 27, 52, and 27 rules respectively. The inputs of these systems were the changes of the prices of the competitor products and their outputs were the changes of the sales of our company. A fourth fuzzy expert system containing 81 rules was used to compensate for the intra-portfolio effects of the price changes of our different models. All the fuzzy rules used were obtained by commonsense reasoning.

6.3. Flow of Data in the System

Given the price changes of the competitor products, various scenarios are automatically created that contain different price changes for our three portfolio products. This way we have explicit control over the resolution of our investigation by producing smaller or greater amounts of such scenarios.

These scenarios are fed both to the three fuzzy expert systems which produce estimates of the sales changes for each product and to the fourth fuzzy expert system which compensates for the intra-portfolio effects. All fuzzy systems’ inputs and outputs are actually percentages of changes making the system insensitive to absolute figures.

After merging all the resulting input-output pairs into a single array we may apply the restrictions so as to exclude combinations that do not respect our strategic constraints.

Then through a linear programming procedure, we may evaluate each of the remaining combinations according to the desired criterion and select the ones of maximum fitness to be presented to the user.

6.4. Numerical Results

Three hypothetical cases were addressed:

1. None of the competitors changes its products’ prices.
2. The price of Mazda MX-3 increased from £14,855 to £15,300 (3%). Some changes are also made to the strategic constraints of our company.
3. The price of Celica 2.0 GT is increased from £18,392 to £18,852 (2.5%). The constraints are the same as for the second case.

Table 2 summarizes both the results of FUZZYPRICE and of PRICESTRAT.

Table 2. *Simulation results for the three cases*

	Case 1		Case 2		Case 3	
	FP	**PS**	**FP**	**FP**	**FP**	**FP**
Price for 8v	15,577	15,655	15,733	15,826	15,577	15,655
Price for 16v	17,303	17,155	17,303	16,826	17,303	
Price for 4×4	21,788	21,655	21,788	21,380	21,788	21,380
Sales for 8v	1051	1036	1065	1031	1051	1030
Sales for 16v	1348	1381	1348	1459	1370	1507
Sales for 4×4	283	286	283	290	283	290
Total profit	21,018,000	21,047,152	21,284,000	21,280,229	21,199,000	21,200,852

All prices and profits in pounds (£). FP: FUZZYPRICE. PS: PRICESTRAT

7. Conclusions and Future Work

FUZZYPRICE is a hybrid fuzzy expert system that is intended to assist a manager for pricing decision making. It has the ability to embed the experience of several human experts in the form of linguistic rules to formulate a knowledge base that can be fine-tuned using historical or market-research data. These are then used systematically to provide decision support for pricing. Its scenario-oriented nature also makes it a good candidate for sensitivity and profit margin analyses.

It is noteworthy that the above results were obtained using only commonsense fuzzy rules and with no historical data to tune them. We have every reason to believe that although FUZZYPRICE is quite simple and elasticities and other important features of the market are rather implicitly than explicitly defined in it, by acquiring valid fuzzy rules from human experts and making use of historical data to fine-tune them could result in a much more robust and successful system.

Possible improvements of FUZZYPRICE include but are not limited to:

- weighting of each competitor's actions (according to their market share or any other criterion),
- weighting of each rule according to its importance (which could take place automatically through reinforcement learning techniques),
- automatically extracting rules based on historical data and/or genetic techniques,
- extending the system's architecture so as to include other factors (e.g. advertisement)

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